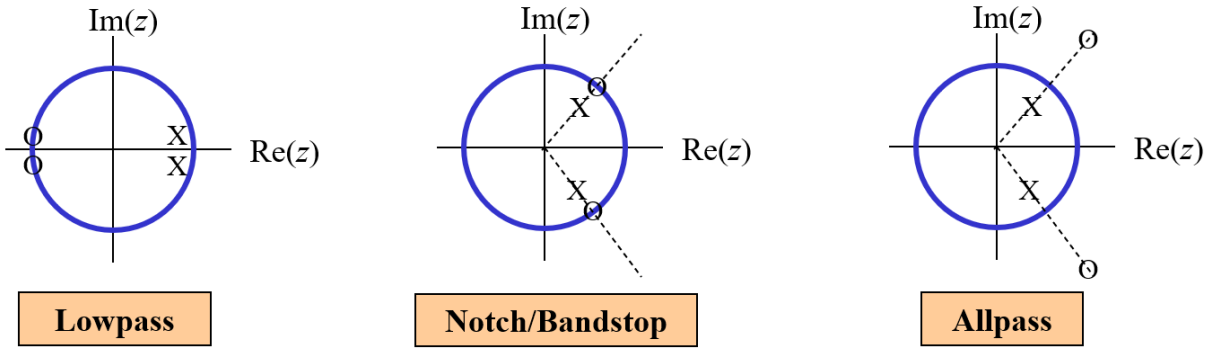
[10:30am] Takeaways from last lectures

- Like FIR filters, IIR filters can be used to achieve a desired arbitrary frequency selectivity
  - Lowpass (moving average and exponential moving average)
  - Bandpass (2nd order bandpass seen on homework, chromagram, vocoder)
  - Highpass (approximation to derivative seen on homework)
  - Notch (Tinnitus filter design problem on homework)
  - A second order (biquadratic) IIR filter is sufficient to design all of the above filter types
- IIR filters may be BIBO unstable
  - Poles must be inside unit circle for stability
  - Implementation could slightly move location of poles, causing instability
  - IIR filters can usually meet specifications using a lower order than FIR

[10:45] Biquad frequency responses

- When designing a second order IIR filter by placing poles and zeros, we can also add an addition gain to the system C
- We can solve for C based on the desired frequency selectivity
  - Example: If we want a lowpass filter, then the frequency response at zero hertz should have a magnitude of one
  - Set $H(\omega = 0) = 1$ and solve for C
- The angle ω on the z plane corresponds to the frequency ($z = e^{j\omega}$)
  - A pole at angle ω will amplify frequency components near ω
  - A zero at angle ω will attenuate frequencies components near ω



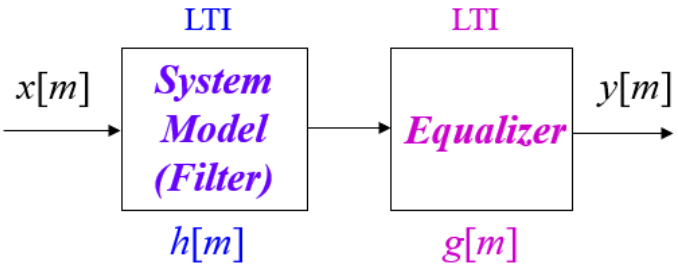
[11:20] Allpass filters

- Handout on allpass filters
- Two types of all-pass filters
  - Pole and zero placed at same location inside unit circle
  - Pole inside unit circle at radius r and angle ω, zero at radius $1/r$ and radius ω

[11:30] Stability and Region of convergence (ROC)

- For causal sequence, poles must be inside the unit circle for stability
- More generally, the region of convergence must include the unit circle
- Three uses of Region of convergence
  - ROC tells us about BIBO stability
  - ROC tells us when we can convert from z-domain to frequency domain
  - ROC tells us how do inverse z transform

[11:40] Equalization



$$Y(z) = H(z)\,G(z)\,X(z)$$

$$\frac{Y(z)}{X(z)} = H(z)\,G(z)$$

- How to design equalizer $G(z)$ that corrects for $H(z)$
  - Approach #1
    - Place poles of G wherever there is a zero of H
    - Place zeros of G wherever there is a pole of H
    - Fails if H has a zero outside unit circle, since G would become unstable
  - Approach #2
    - Place poles of G at same angle as zero of H, but with reciprocal radius
    - Use to fix the failure mode of approach #1
  - Approach #3
    - If H has a zero on the unit circle ($r = 1$), #1 and #2 both fail
    - The best we can do is place a pole at radius $r = 1 - \epsilon$ for some very small value of ϵ, e.g. $r = 0.98$.

[11:45] Linear phase

- Linear phase is very important for some applications, but not all.
- Image processing demo on the effect of phase